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ORIGINAL RESEARCH

The industrial production index of Japan and South Korea: a dynamic relationship

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Abstract In this paper, a dynamic relationship between the industrial production index (IPI) of Japan and the IPI of South Korea is presented. With the help of a VAR(2) model, and using the terminology of Granger causality, it is shown that the IPI of Japan Granger causes the IPI of South Korea, but the IPI of South Korea Granger does not cause the IPI of Japan. Other aspects of this dynamic relationship between these two indices are presented as well.

Keywords Industrial production index (IPI) · VAR models · Granger causality · Impulse response functions · Forecast error variance decomposition · Software: R, MTS, RATS, vars

Introduction

The industrial production index (IPI) is an index covering production in mining, manufacturing, and public utilities (electricity, gas, and water), but excluding construction.

Production indices are normally compiled at monthly or quarterly frequency to measure increases and decreases in production output. The final focus is the compilation of annual statistics. Indices of industrial production that is compiled in all OECD member countries are used as a main short-term economic indicator in their own right because of the impact that fluctuations in the level of industrial activity have on the remainder of the economy.

In this paper, we consider a dynamical relationship between the IPIs of Japan and South Korea with the help a vector autoregressive model of order two: VAR(2).

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VAR(p) models

According to Tsay (2014, pp. 27–30), the most commonly used econometric model for multiple time series is the multiple vector autoregressive or VAR model, and there are good reasons for this assertion.

The multivariate time series z_t follows a VAR model of order p if it can be written as follows:

$$z_t = \phi_0 + \sum_{i=1}^{p} \phi_i z_{t-i} + a_t$$

with z_t a vector of time series of dimension $k \times 1$, ϕ_0 a vector of constants of dimension: $k \times 1$, ϕ_i a matrix of dimension $k \times k$, for $i > 0$ and $\phi_p \neq 0$, and a_t a sequence of independent and identically distributed (i.i.d.) random vectors with mean zero and co-variance matrix Σ_a positive definite.

To summarize the properties of VAR(p) models, it is interesting to start with the bilinear VAR(1):

$$z_t = \phi_0 + \phi_1 z_{t-i} + a_t$$

written explicitly as:

$$\begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} = \begin{pmatrix} \phi_{10} \\ \phi_{20} \end{pmatrix} + \begin{pmatrix} \phi_{1,11} & \phi_{1,12} \\ \phi_{1,21} & \phi_{1,22} \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \end{pmatrix}$$

or

$$z_{1t} = \phi_{10} + \phi_{1,11} z_{1,t-1} + \phi_{1,12} z_{2,t-1} + a_{1t}$$

$$z_{2t} = \phi_{20} + \phi_{1,21} z_{1,t-1} + \phi_{1,22} z_{2,t-1} + a_{2t}$$

That is, $\phi_{1,12}$ shows the linear dependence of z_{1t} on $z_{2,t-1}$ in the presence of $z_{1,t-1}$. And $\phi_{1,21}$ measures the linear relationship between z_{2t} and $z_{1,t-1}$ in the presence of $z_{2,t-1}$ and similarly with the other coefficients of matrix ϕ_1.

This matrix representation of the model gives us the insight of the so-called Granger causality.

If the off-diagonal elements of matrix ϕ_1 are zero, that is $\phi_{1,12} = \phi_{1,21} = 0$, the consequence is that z_{1t} and z_{2t} are not dynamically correlated, in which case each series follows a univariate AR(1) model, which can be handled accordingly. The two series are said to be uncoupled.

On the other side, if $\phi_{1,12} = 0$ but $\phi_{1,21} \neq 0$ then we have

$$z_{1t} = \phi_{10} + \phi_{1,11} z_{1,t-1} + a_{1t}$$

and

$$z_{2t} = \phi_{20} + \phi_{1,21} z_{1,t-1} + \phi_{1,22} z_{2,t-1} + a_{2t}$$

That is, z_{1t} does not depend on the past values of z_{2t} but z_{2t} depends on the past values of z_{1t}. This is an example of a transfer function relationship, in control engineering. In econometrics, this is an example of Granger causality between two series, with z_{1t} causing z_{2t} but z_{1t} not being caused by z_{2t}.

As Tsay (2014, p. 29) remarks for this bivariate VAR(1) model, if the variance–covariance matrix Σ_a is not diagonal, then z_{1t} and z_{2t} are instantaneously or contemporaneously correlated, with instantaneous Granger causality, going in both directions.

The Data

From the database of the Federal Reserve Bank of St. Louis, we obtained the data for the industrial production indices of Japan and South Korea, taken from the Main Economic Indicators-complete database, a publication of the Organisation for Economic Co-operation and Development (OECD), accessed on 10-14-2014. In this paper, we consider the IPI for both countries as annual data and not seasonally adjusted.

Due to the different starting years of the index data, here we use data from 1980 to 2013; a total of 33 observations. The last six observations of these series are as follows:

	Japan	SKorea
28	113.77411	83.28158
29	110.13249	86.08375
30	86.97609	85.97600
31	97.07524	105.95833
32	97.70852	107.42500
33	96.85860	107.74167

Notice the different behaviors of Japanese and South Korean data at the end of this sample.

Using the RATS package, the graphic evolution of these two series appears in Fig. 1.

At first look, the two series seem to be non-stationary, but with VAR models their transformation is not recommended (RATS, User's Guide, p. 205).

The basic statistics, mean, and standard deviation of both series are as follows:

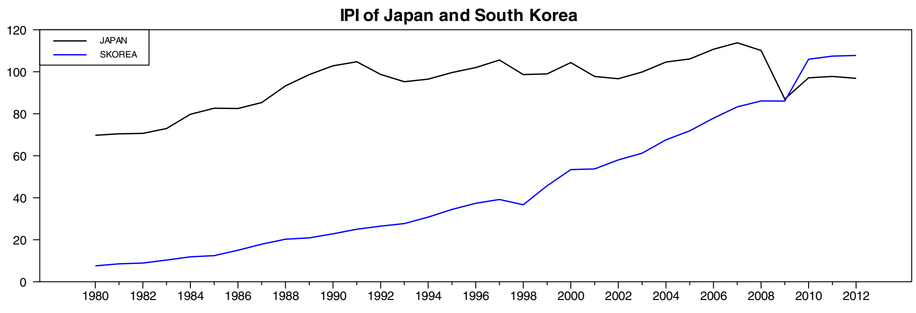

Fig. 1 Temporal evolution of Japanese IPI and South Korean IPI : 1980–2013

Mean:	
Japan	SKorea
94.86357	44.83992
Standard deviation:	
Japan	SKorea
11.98977	31.37950

In our case, the bivariate vector z_t is

$$z_t = \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix}$$

with Japan's IPI as z_{1t} and South Korea's IPI as z_{2t}.

Next, the number of lags of the model has to be determined. With the help of RATS package, and using the Akaike information criterion (AIC), corrected for degrees of freedom, the order of our VAR is

VAR lag selection	
Lags	AICC
0	16.0209602
1	11.3257987
2	11.2212767*
3	11.2655814
4	11.6957794
5	12.3560570
6	13.2148853
7	14.7279664
8	16.6339483

That is, $p = 2$. With this value of p, we come to the MTS package for the estimation of the model *m1*.

```
m1 = VAR(datos,2)
Constant term:
Estimates:  21.19133 2.832771
Std.Error:  10.26493 4.897367
AR coefficient matrix
AR( 1 )-matrix
       [,1]  [,2]
[1,]  0.698 0.255
[2,] -0.546 1.264
standard error
      [,1]  [,2]
[1,] 0.234 0.373
[2,] 0.112 0.178
AR( 2 )-matrix
       [,1]   [,2]
[1,] 0.0902 -0.275
[2,] 0.5433 -0.259
standard error
      [,1]  [,2]
[1,] 0.241 0.408
[2,] 0.115 0.194

Residuals cov-mtx:
         [,1]      [,2]
[1,] 28.07192 10.316474
[2,] 10.31647  6.389766

det(SSE) =  72.94335
AIC =  4.774532
BIC =  5.137321
HQ  =  4.896599
```

This estimated model has non-significant coefficients at the usual $\alpha = 0.05$ significance level. Suppressing these coefficients, we get the simplified *m1simplif* model:

```
m1simplif = refVAR(m1,thres=1.96)
Constant term:
Estimates:  21.82964 0
Std.Error:  8.470252 0
AR coefficient matrix
AR( 1 )-matrix
       [,1] [,2]
[1,]  0.781 0.00
[2,] -0.431 1.02
standard error
       [,1]   [,2]
[1,] 0.0880 0.0000
[2,] 0.0847 0.0194
AR( 2 )-matrix
      [,1] [,2]
[1,] 0.000    0
[2,] 0.459    0
standard error
       [,1] [,2]
[1,] 0.0000    0
[2,] 0.0868    0

Residuals cov-mtx:
         [,1]     [,2]
[1,] 28.57840 10.80264
[2,] 10.80264  6.99180

det(SSE) =  83.1174
AIC =  4.662678
BIC =  4.844073
HQ  =  4.723712
```

Now, this model has significant coefficients, and validating residuals, and in a different format, we can write:

$$\begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} = \begin{pmatrix} 21.83 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.781 & 0 \\ -0.431 & 1.02 \end{pmatrix} \begin{pmatrix} z_{1,t-1} \\ z_{2,t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0.459 & 0 \end{pmatrix} \begin{pmatrix} z_{1,t-2} \\ z_{2,t-2} \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \end{pmatrix}$$

With a variance–covariance matrix of residuals:

$$\hat{\Sigma}_a = \begin{pmatrix} 28.578 & 10.803 \\ 10.803 & 6.992 \end{pmatrix}$$

Written separately, the two estimated models, are as follows:

$$\hat{z}_{1t} = 21.83 + 0.781 z_{1,t-1}$$

and

$$\hat{z}_{2t} = -0.431 z_{1,t-1} + 1.02 z_{2,t-1} + 0.459 z_{1,t-2}$$

In words, the IPI of Japan, in the presence of South Korea's IPI, depends on its lagged first period, while the IPI of South Korea in the presence of Japan's IPI depends on its first lagged period and on the two lagged periods of the Japanese IPI. If we employ the terminology of Granger causality, we can say that the IPI of Japan causes the IPI of South Korea, but the IPI of South Korea does not cause the IPI of Japan.

The impulse response

The VAR(p) models allow us to establish the dynamical relationship between the variables of the system, but at the same time it is possible to consider this relationship from other points of view: the impulse response and the forecast error variance decomposition.

Sometimes it is of interest to evaluate the effects of a stochastic change or shock in one variable on its own evolution and on the evolution of the others variables. This type of analysis is known as the impulse response or multiplier analysis.

The impulse response measures the effect of a shock caused in one variable of the system on its own and on the rest of the variables. This effect it is better understood in the MA version of the VAR model.

$$z_t = \mu + a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} + \theta_3 a_{t-3} + \cdots$$

truncated at some lag, q, with $\theta_0 = 1$ and a_t a succession of i.i.d. random innovations, having mean zero and constant variance.

In compact form:

$$z_t = \mu + \sum_{i=0}^{q} \theta_i a_{t-i};\ \theta_0 = 1$$

If we now induce a unitary impulse or a shock in a_t; that is, if we set to zero all the a_t except one, a_{t-k}, which is set to 1, then by successive substitutions, we get

$$\begin{aligned} z_{t-k} - \mu &= \theta_{t-k} \\ z_{t-k-1} - \mu &= \theta_{t-k-1} \\ &\vdots \\ z_t - \mu &= \theta_t \end{aligned}$$

This succession of values of θ_i is called the impulse response on z_t of the unitary shock in a_t.

If Σ_a is not diagonal, it is unrealistic to consider that a unitary shock induced in the error term of one of the variables in the VAR can be isolated from the other errors terms. That is it would be impossible to establish the impact of a unitary shock in the IPI of Japan on the IPI of South Korea. To solve this problem we could use the Cholesky decomposition of matrix Σ_a, a solution possible due to the fact that this matrix is symmetric and positive definite. In this case, there is a matrix P such that $\Sigma_a = PP$' and $P'\Sigma P'^{-1} = I$. With this P^{-1} it is possible to convert a_t on a vector of uncorrelated errors e_t, that is

$$z_t = \mu + \sum_{i=0}^{q} \theta_i P P^{-1} a_{t-i} = \mu + \sum_{i=0}^{q} B_i e_{t-i}$$

after substituting $B_i = \theta_i P$ and $e_t = P^{-1} a_t$. The elements of B_i are the impulse response coefficients of z_t with orthogonal innovations.

The problem involved in the Cholesky decomposition of Σ_a should be mentioned. That is, the order of variables in the vector z_t has consequences. This is not the place for more details, but we could consider this artificiality as the cost for clarifying the impulse response of the system to the new uncorrelated e_t.

Coming to our case, and using the software RATS, the impulse response of a unitary shock in the IPI of Japan is

Responses to shock in Japan

Entry	Japan	SKorea
1	5.34587718	2.0207426
2	4.17300936	−0.2401093
3	3.25746487	0.4095402
4	2.54278782	0.9288720
5	1.98490856	1.3467273
6	1.54942617	1.6856245
7	1.20948718	1.9631471
8	0.94412968	

And the impulse response to a shock in the IPI of South Korea:

Responses to shock in SKorea

Entry	Japan	SKorea
1	0.00000000	1.7054029
2	0.00000000	1.7402933
3	0.00000000	1.7758974
4	0.00000000	1.8122300
5	0.00000000	1.8493059
6	0.00000000	1.8871403
7	0.00000000	1.9257487
8	0.00000000	1.9651471

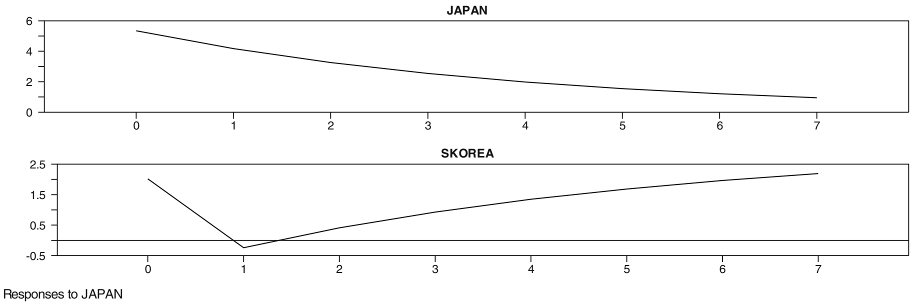

Fig. 2 Impulse response in Japanese IPI

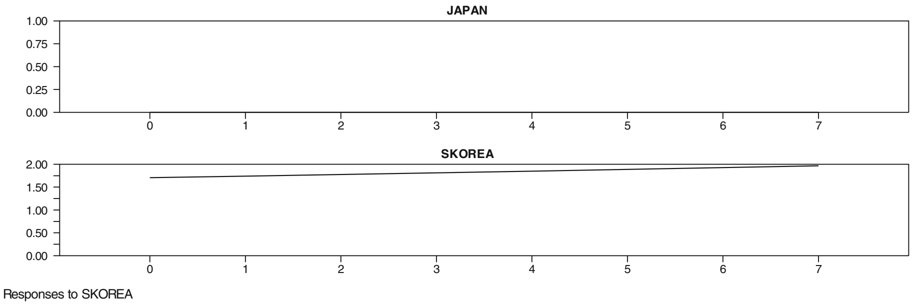

Fig. 3 Impulse response in South Korean IPI

The graphical representation of these responses is depicted in Figs. 2 and 3.

In Fig. 2, the unitary impulse induced in the Japanese IPI decreases with the passing of time, but has a growing impact in the IPI of South Korea. In contrast, as shown in Fig. 3, the unitary impulse induced in the IPI of South Korea has an effect only on its own IPI.

The forecast error variance decomposition

Another point of view for the dynamical relationship established by a VAR model is given by the forecast error variance decomposition (FEVD), which allows us to assign the fraction of the variance of the error due to each of the variables in the system. In other words, this decomposition allows us to attribute the error variance at the sources.

Again with RATS, this is the decomposition in our case (Tables 1, 2):

In the first column of these tables are the estimated standard errors of the predictions, here to a horizon of 8 periods (years). The other columns contain the percentage of the variance due to each of the variables. The sum of each row adds to the total. For the first table, 100 % of the error variance is due to Japan. In the second table, the percentage of the error variance is more distributed, with South Korea having the major part.

Table 1 Decomposition of variance for series Japan

Step	SE	Japan	SKorea
1	5.34587718	100.000	0.000
2	6.78177041	100.000	0.000
3	7.52352891	100.000	0.000
4	7.94161553	100.000	0.000
5	8.18590979	100.000	0.000
6	8.33125684	100.000	0.000
7	8.41859251	100.000	0.000
8	8.47136829	100.000	0.000

Table 2 Decomposition of variance for series SKorea

Step	SE	Japan	SKorea
1	2.64420119	58.403	41.597
2	3.17459811	41.090	58.910
3	3.66054750	32.156	67.844
4	4.18886486	29.473	70.527
5	4.77286029	30.664	69.336
6	5.40211290	33.672	66.328
7	6.06178840	37.231	62.769
8	6.73917240	40.712	59.288

Conclusion

In this paper, a dynamic relationship between the IPI of Japan and the IPI of South Korea has been established. With the presented model, the Granger causality principle allows us to say that the IPI of Japan causes the IPI of South Korea. However, the Granger causality principle is a statistical concept, useful for forecasting time series. As a consequence, it cannot be extrapolated directly to represent the relationship of the Japanese and South Korean industries. This is a relationship to be explained by structural studies, far beyond the objective of this paper.

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