

# Optimal Environmental Policy in a Small Open Economy Under a New Keynesian Framework

Yongseung Jung<sup>1</sup> · Doo Yong Yang<sup>2</sup>

Received: 8 September 2023 / Accepted: 20 Octboer 2023 /

Published online: 1 December 2023

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**Abstract** This paper analyzes the optimal environmental policy under the new Keynesian environment. First, this paper conducts an empirical analysis using panel VAR. The results show that a 10% reduction in carbon dioxide is associated with a 2% reduction in GDP. To build a theoretical model, we next model the impact of the environment on economic activity in a small open economy and analyze the impact of environmental policies under a new Keynesian model. The results are as follows. First, optimized taxation should respond strongly to emissions and output since the emission is procyclical and firms' abatement efforts are procyclical, dampening business cycle fluctuations. Second, environmental taxation should be levied on domestic emission flows, not the overall stock of emissions, since it is more efficient and likely to generate less macroeconomic volatility by inducing firms to reduce emissions and optimally adjust their activities over time. Finally, the simple optimized environmental taxations show that the environmental taxation policy should be aggressive and not tax-smoothing type since the so-called tax-smoothing policy is welfare detrimental.

**Keywords** Environment Policy · Panel VAR · New Keynesian Model

**JEL Classification** E52 · F31

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✉ Yongseung Jung  
jungys@khu.ac.kr

Doo Yong Yang  
yangdy@khu.ac.kr

<sup>1</sup> Kyung Hee University.

<sup>2</sup> Corresponding Author, Kyung Hee University.

## Introduction

Climate change is the biggest challenge in economics today. Production requires energy obtained by burning fossil fuels, and large-scale pollution occurs. It causes unnatural environmental disasters by accumulating greenhouse gases in the atmosphere and increasing the global temperature. Such changes in global temperature negatively affect the economic production process, limiting economic activities in various fields such as production, consumption, and investment. In this way, environmental and economic activities mutually influence (Nordhaus 1975, 1977).

Considering the environment from an economic point of view in the early days is the question of whether the economy can grow continuously without being restricted by natural resources or whether economic growth will shrink in the long run because economic growth causes irreversible environmental pollution. As economic activity increases, energy and resource consumption rapidly produce contaminants. Increased extraction of natural resources, accumulation of waste, and increased concentrations of pollutants may exceed the sustainable capacity of ecosystems, reducing environmental quality and long-term social well-being despite increased incomes. Against this background, to preserve the environment and sustain economic activity, it is important to restrain economic growth to some extent and maintain an appropriate economic growth process, including the environment (Stokey, 1998).

On the other hand, some argue that economic growth further expands awareness of the environment, making environmental preservation a more important issue. An increase in income raises concerns about resource depletion and awareness of environmental pollution control, thereby increasing the demand for factors of production with low resource consumption, improving the quality of the environment, and expanding environmental conservation measures.

However, such environmental and economic viewpoints have recently shifted away from the existing naive viewpoint to the position that environmental destruction threatens the entire economic system without extreme measures. In 2021, the IPCC urged a strong greenhouse gas reduction policy to keep the global temperature within 1.5 compared to the pre-industrial period. It is a policy to not cross the irreversible tipping point due to global warming. Accordingly, governments in developed countries and other countries have proposed reduction limits for carbon neutrality by 2050. Carbon neutrality is a state of net-zero carbon dioxide emissions. Korea also proposed a 40% carbon reduction plan by 2030 compared to 2018. Environmental measures like these will have a major impact on the economy. In the case of China, if the proposed target is implemented by 2030, economic growth is predicted to decrease by 1.7-2% per year. (Nordhaus 2019). Such changes in global environmental policy will act as a global shock to the entire economy.

Against this background, this paper aims to analyze the impact of such environmental policy changes on the economy. In particular, we will analyze how each environmental policy affects the macroeconomy under a new Keynesian model. The major contribution of the paper is to determine the most optimal policy for a small open economy. Studies on the combination of macroeconomic models and ecosystems have been continuously conducted. Using a neoclassical growth model, Nordhaus (1977) showed how the environment influences long-term economic growth. In particular, he modeled the pathways of how carbon emissions in the geophysical sector lead to the greenhouse effect and how this affects atmospheric temperature and thus

modeled the economic impacts. The case of modeling the relationship between long-term economic growth and the environment using the neoclassical growth model was continued by Seldon and Song (1995) and Stokey (1998). Heutel (2012) analyzed the impact of the environment on the economy by including pollution externality in a dynamic stochastic general equilibrium real business cycle model. Annicchiarico and Di Dio (2015) analyzed the impact of environmental policy in the presence of nominal and real uncertainty by grafting environmental factors into a new Keynesian model.

This paper is composed as follows. In section 2, We describe the empirical analysis. Section 3 specifies a small open economy RANK model and derives an equilibrium. In section 4, we discuss the implications of the model with environmental policy and the quantitative implications of the model. Finally, we give concluding remarks in section 5.

## Empirical Analysis

This chapter empirically analyzes the impact of carbon emissions on key macroeconomic factors. To this end, carbon intensity was first examined, which shows the relationship between economic production and pollution. Carbon intensity is the ratio of GDP to CO<sub>2</sub> emissions. In other words, high carbon intensity means much pollution is emitted in production.

Figure 1 shows the global carbon intensity from 1960 to 2018. Since 1960, carbon intensity has decreased by an average of 1.2% annually. Although carbon emissions are continuously decreasing in the production process, it is a recent evaluation that it cannot reduce or eliminate unusual disasters (Nordhaus, 2019). In particular, the consensus is that past global environmental policies did not significantly contribute to curbing global warming. It is convincing that the Kyoto Protocol in 1997 or the Paris Accord in 2015 did not lead to a sharp decrease in carbon intensity.

In the case of developed countries, it is shown that carbon generated in the production process is relatively small. The scale of CO<sub>2</sub> emissions in developed countries is 42% smaller than the global average. What is unusual is that the gap between these developed countries has remained the same since 1960.

## Panel VAR Model Specification

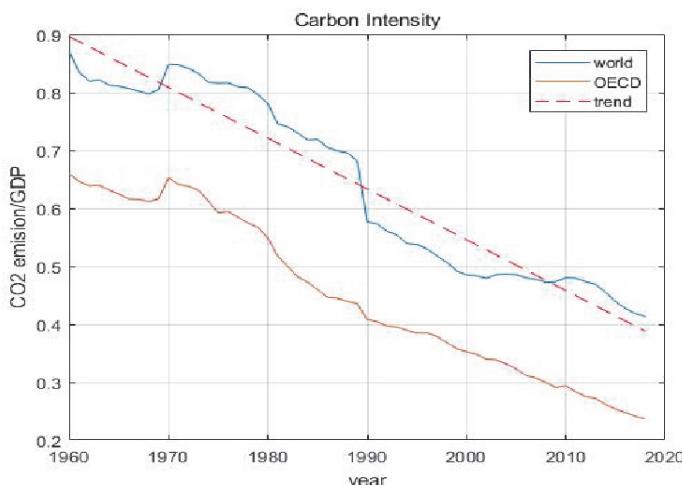
The interactions between pollution and macroeconomic variables are analyzed using a panel VAR framework. It accounts for individual country heterogeneity while allowing for dynamic relationships between multiple endogenous variables. The model is expressed as follows:

$$Y_{it} = \mu_i + A(L)Y_{it} + \epsilon_{it} \quad (1)$$

where  $Y_{it}$  is a  $GN \times 1$  vector of  $G$  number of endogenous variables,  $i (i = 1, 2, \dots, N)$  denotes the country, and  $t$  is the time.  $A(L)$  is a matrix in the lag operator  $L$ .  $\epsilon_{it}$  is a vector of error terms. Two main restrictions characterize this specification. First, it assumes common slope coefficients. Second, it does not allow for interdependencies across countries. With these

restrictions, the interest is typically in estimating the average dynamics in response to shocks. The endogenous vector is composed of following variables where  $\mathbf{Y}_{it} = [\mathbf{GDP}, \mathbf{CONSUM}, \mathbf{INV}, \mathbf{GOV}, \mathbf{CO2}]$  consists of output ( $\mathbf{GDP}$ ), consumption ( $\mathbf{CONSUM}$ ), investment ( $\mathbf{INV}$ ), and CO2 emission ( $\mathbf{CO2}$ ).

Given a relatively small number of observations, we estimate the model via Bayesian panel VAR. The Bayesian allows us to estimate a model with a limited sample, avoiding over parameterization.

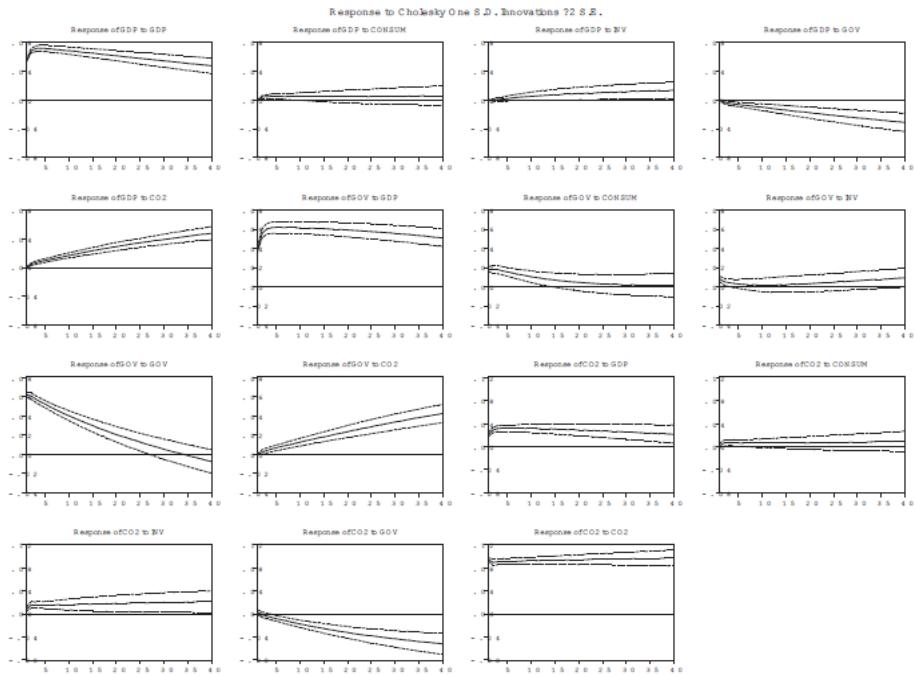


Source: World Development Indicator (World bank)

**Fig. 1** Trend in Global Carbon Intensity

## Results from Panel VAR Estimation

Impulse response functions (IRFs) derived from the estimation of (1) show the dynamic behavior of endogenous variables for various shocks. Figure 2 displays the IRFs for the whole panel. A positive GDP shock generates a positive response to consumption and investment. On the other hand, a GDP shock creates a negative response to government spending. Interestingly, the GDP shock acts as a permanent shock on carbon emissions. It means that temporary GDP shocks have permanent consequences for carbon emissions. A government spending shock produces a response similar to a GDP shock. A positive carbon emission shock induces a positive response to output, consumption, and investment. Of particular note, if a carbon emission shock of 10% occurs, GDP increases by 2%. Since the IRF is symmetrical, a 10% reduction in carbon emissions will result in a 2% reduction in GDP.



Source: authors' calculation

**Fig. 2** Impulse Response Function

## Model

### Households

Households choose their consumption, asset holdings, and labor supply to maximize their expected lifetime utility function subject to a sequence of budget constraints:

$$E_t [\sum_{i=0}^{\infty} \beta^i u(C_{t+k}, N_{t+k})], \quad 0 < \beta < 1, \quad (2)$$

where  $u(C_{t+k}, N_{t+k}) = \frac{C_{t+k}^{1-\sigma}-1}{1-\sigma} - \frac{N_{t+k}^{\sigma_1+\nu}}{1+\nu}$  ( $\sigma \neq 1$ ),  $\beta$  is the household's discount factor, and  $E_t \equiv \sum x_{t+1} f(x^{t+1}|x^t)$  denotes the mathematical expectation operator over all possible states of nature on history  $x^t$ . Here  $x^t = \{x_0, \dots, x_t\}$  denotes the history of events up to period  $t$ .  $C_{t+k}$  and  $N_{t+k}$  represent domestic household's consumption, total working hours in period  $t+k$ , respectively.  $C_t$  is a composite consumption index defined by

$$C_t = [(1-\theta)^{\frac{1}{\psi}} C_{H,t}^{\frac{\psi-1}{\psi}} + \theta^{\frac{1}{\psi}} C_{R,t}^{\frac{\psi-1}{\psi}}]^{\frac{1}{\psi}}, \quad \psi > 0. \quad (3)$$

Here  $C_{H,t}$  and  $C_{F,t}$  are indices of domestic and foreign consumption goods of asset holders, and  $\theta$  represents the share of imported goods in domestic household's consumption. The indices are given by the following CES aggregator of the quantities consumed of each variety of good:

$$C_{H,t} = \left[ \int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad C_{F,t} = \left[ \int_0^1 C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1. \quad (4)$$

Here  $\psi$  and  $\epsilon$  measure the elasticity of substitution between domestic and foreign goods, and the elasticity of substitution among goods within each category, respectively.

Assume that there is a domestic currency-denominated bond market. When there is a complete asset market,  $B_t$  is the one-period nominal contingent home-currency bond purchased in period  $t$  and  $\Delta_{t,t+1}$  is the corresponding stochastic discount factor in period  $t$ . The riskless one-period nominal interest rate in period  $t$  is given by  $R_t \equiv [E_t \Delta_{t,t+1}]^{-1}$ . The domestic household receives lump-sum transfers from the government, and wages, rents for capital, and dividends from each firm. Then the household's budget at the beginning of the period  $t$  is given by

$$P_t(C_t + I_t) + E_t[\Delta_{t,t+1}B_t] \leq B_{t-1} + W_t N_t + V_t K_t + D_t + T_t, \quad (5)$$

where  $P_t$  is the home currency price of goods in period  $t$ . Here  $T_t$ ,  $D_t$ ,  $W_t$ ,  $V_t$  denote the domestic household's lump-sum transfer or tax, nominal dividends and wages from domestic firms, and nominal rental rates for capital stock given to the home residents, respectively.

Assume that a household in each country owns only its own country's capital stock to rent to its country's firm, and there is no firm-specific capital stock. Since we do not empirically observe large discrete capital stock adjustments, it is reasonable to introduce an adjustment cost in capital stock installments. If there are costs of installing capital, the capital stock will move more sluggishly. To preserve the simple model structure as far as possible, the Christiano et al. (2005) type investment adjustment cost is adopted as follows:

$$K_{t+1} = (1 - \delta_k)K_t + (1 - F(I_t/I_{t-1}))I_t, \quad (6)$$

where  $F(I_t/I_{t-1})$  is a positive function of changes in investment as in Christiano et al. (2005). In particular,  $F = F' = 0$  at the steady-state, and  $F'' > 0$ .  $I_t$  is the composite investment of the home household at period  $t$ , and  $K_t$  is the composite capital stock of the home household at period  $t$ .

### First Order Conditions

First order conditions for asset holders can be summarized as follows.

$$C_t^{-\sigma} = \Lambda_t, \quad (7)$$

$$N_t^v = \Lambda_t w_t, \quad (8)$$

$$\Delta_{t,t+1} = \beta \frac{\Lambda_t}{\Lambda_{t+1}} \quad (9)$$

$$Q_{k,t} \Lambda_t = \beta E_t [\Lambda_{t+1} (V_{t+1} + Q_{k,t+1} (1 - \delta))], \quad (10)$$

$$K_{t+1} = (1 - F(I_t/I_{t-1})) I_t + (1 - \delta_k) K_t, \quad (11)$$

$$Q_{k,t} [F(I_t/I_{t-1}) + F'(I_t/I_{t-1}) I_t/I_{t-1}] - \beta E_t [\frac{\Lambda_{t+1}}{\Lambda_t} Q_{k,t+1} F'(I_{t+1}/I_t) (I_{t+1}/I_t)^2] = 1, \quad (12)$$

and the budget constraint (5). Here  $\Lambda_t$  is the Lagrange multiplier of the budget constraints and  $v_t = \frac{V_t}{P_t}$ ,  $w_t = \frac{W_t}{P_t}$ . Equation (7) is the first order conditions for consumption goods, and (8) relates the marginal disutility of labor hours to the marginal utility of the real wage rate. Equation (9) and (10) refer to the intertemporal decision of the domestic asset holder, that is, the decision of bond holdings and capital stock holdings, respectively. Equation (12) represents the relationship between the rent paid to a unit of capital in  $t+1$  and the expected return to holding a unit of capital from  $t$  to  $t+1$  and thus the evolution of Tobin's  $q$  over time.

Under the assumption of complete asset markets, an optimal risk sharing condition implies that the marginal utility of consumption of foreign household is proportional to the marginal utility of domestic household, i.e.

$$\left(\frac{c_t}{c_t^*}\right)^\sigma = q_t, \quad (13)$$

where  $q_t \equiv \frac{\varepsilon_t p_t^*}{p_t}$  is the real exchange rate, and  $\varepsilon_t$  is the nominal exchange rate in period  $t$ . The foreign values of the corresponding domestic variables will be denoted by an asterisk (\*). The real exchange rate given by (13) says that the real value of one unit of domestic currency in domestic market,  $\frac{c_t^\sigma}{p_t}$  should be equal to the real value of the corresponding unit in foreign currency in foreign market,  $\frac{(c_t^*)^{-\sigma}}{\varepsilon_t p_t^*}$ .

## Firms

There are two types of firms in each country. A continuum of monopolistically competitive firms indexed by  $i$ ,  $0 \leq i \leq 1$ , each of which produces its differentiated intermediates  $y_t(i)$ , and a distinct set of perfectly competitive firms, which combine all the intermediate goods into a single final good  $y_t$ .

### Final-Good Firms

The final-good producing firms combine the differentiated domestic intermediate goods  $Y_{H,t}(i)$  using the CES aggregator

$$Y_{H,t} = \left[ \int_0^1 Y_{H,t}(i)^{\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \quad (14)$$

to produce  $Y_{H,t}$  and then combine a composite of foreign intermediate goods  $Y_{F,t}(i)$  given by

$$Y_{F,t} = \left[ \int_0^1 Y_{F,t}(i)^{\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \quad (15)$$

to produce a composite good

$$Y_t = [(1 - \theta)Y_{H,t}^{\frac{1}{\psi}} + \theta Y_{F,t}^{\frac{1}{\psi}}]^{\frac{\psi}{\psi-1}}, \quad \psi > 0. \quad (16)$$

The optimal allocation for each differentiated good yields the demand functions:

$$Y_{H,t}(i) = \left[ \frac{p_{H,t}(i)}{p_{H,t}} \right]^{-\epsilon} Y_{H,t}, \quad Y_{F,t}(i) = \left[ \frac{p_{F,t}(i)}{p_{F,t}} \right]^{-\epsilon} Y_{F,t} \quad (17)$$

for all  $i \in [0, 1]$ , where  $P_{H,t} = \left( \int_0^1 P_{H,t}^{1-\epsilon}(i) di \right)^{\frac{1}{1-\epsilon}}$  and  $P_{F,t} = \left( \int_0^1 P_{F,t}^{1-\epsilon}(i) di \right)^{\frac{1}{1-\epsilon}}$  are the price indexes for domestic and foreign goods, both expressed in home currency.

The optimal allocation of expenditures between domestic and foreign goods implies:

$$Y_{H,t} = (1 - \theta) \left( \frac{p_{H,t}}{p_t} \right)^{-\psi} Y_t, \quad Y_{F,t} = \theta \left( \frac{p_{F,t}}{p_t} \right)^{-\psi} Y_t, \quad (18)$$

Here  $P_t$  is the price index of the final good given by

$$P_t = [(1 - \theta)P_{H,t}^{1-\psi} + \theta P_{F,t}^{1-\psi}]^{\frac{1}{1-\psi}}. \quad (19)$$

### Intermediate-Good Firms

Each intermediate-goods firm  $i$  produces its differentiated output  $Y_t(i)$  with constant returns to scale, concave production technology.

$$Y_t(i) = Z_t K_t(i)^\alpha N_t(i)^{1-\alpha}, \quad (20)$$

where  $Z_t$  is a transitory technology process at period  $t$ ,  $N_t(i)$  is an index of different types of

labor used by the firm  $i$ , and  $\Phi$  is a fixed cost. It is assumed that the total factor productivity includes a damage function  $\Gamma(X_t)$  which decreases with the stock of atmospheric carbon  $X_t$  and the exogenous component of TFP  $A_t$  as follows:

$$Z_t = \Gamma(X_t) A_t, \quad (21)$$

where  $\Gamma(X_t) \equiv 1 - (d_0 + d_1 X_t + d_2 X_t^2)$  and  $\log A_t = (1 - \rho_A) + \rho_A \log A_{t-1} + \varepsilon_{A,t}$ ,  $0 < \rho < 1$ , and  $\varepsilon_{A,t}$  is an i.i.d.  $N(0, \sigma_A^2)$ .

Atmospheric carbon  $X_t$  is fueled by total domestic emissions  $e_t = \int_0^1 e_t(i) di$  and exogenous rest of the world emissions  $e_t^*$ :

$$X_t = \rho_X X_{t-1} + e_t + e_t^*. \quad (22)$$

Firm level emissions are an increasing and concave function of total production

$$e_t(i) = (1 - U_t(i)) \omega_1 Y_t^{1-\omega_2}(i), \quad (23)$$

where  $U_t(i)$  is the fraction of emissions abated by firm  $i$ .

Firm level abatement costs  $\xi_t$  are proportional to production

$$\xi_t(i) = \kappa_1 Y_t(i) U_t(i)^{\kappa_2}. \quad (24)$$

### Price Setting in Intermediate-Goods Sector

Intermediate good firms set prices in the currency of the seller and the Law of One Price holds for analytical simplicity. We assume that changing prices is subject to some cost as in Rotemberg (1982). The cost of adjusting prices for each domestic firm  $i$  equal to

$$\frac{\Theta}{2} \left( \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right)^2 P_{H,t}, \quad (25)$$

where the parameter  $\Theta$  measures the degree of price stickiness. The higher  $\Theta$  the more sluggish is the adjustment of nominal prices. If  $\Theta = 0$ , prices are flexible.

Under the assumption that government levies environmental taxes  $\tau_{e,t}$  on pollution  $e_t(i)$ , the firm  $i$ 's maximization problem can be written as follows:

$$\begin{aligned} \max. E_0 \{ & \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t P_0}{\Lambda_0 P_t} [P_{H,t}(i) Y_t(i) - W_t N_t(i) - V_t K_t(i) - \tau_{e,t} e_t(i) \\ & - \kappa_1 Y_t(i) U_t(i)^{\kappa_2} - \frac{\Theta}{2} \left( \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right)^2 P_{H,t}] \} \end{aligned} \quad (26)$$

subject to

$$\begin{aligned} Y_t(i) &= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t, \\ Y_t(i) &= Z_t K_t(i)^\alpha N_t(i)^{1-\alpha}, \\ e_t(i) &= (1 - U_t(i)) \omega_1 Y_t^{1-\omega_2}(i). \end{aligned}$$

$D_t = \{P_{H,t}(i), Y_t(i), N_t(i), K_t(i), \Gamma_t(i), \Theta_t(i)\}$  represents the choice variables by firm  $i$  at time  $t$ . Firm's first order conditions are given by

$$V_t = MC_t(i) \frac{Y_t(i)}{K_t(i)}, W_t = MC_t(i) \frac{Y_t(i)}{N_t(i)}, \quad (27)$$

$$\begin{aligned} \frac{\theta}{P_{H,t-1}(i)} \left( \frac{P_{H,t}(i)}{P_{H,t-1}(i)} - 1 \right) P_{H,t} &= \{ (1 - \varepsilon) \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t \\ &\quad + \varepsilon \omega_2 \tau_{s,t} (1 - \chi_t(i)) \omega_1 \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon(1-\omega_2)-1} Y_t^{1-\omega_2} \\ &\quad + \kappa_1 \varepsilon U_t(i)^{\kappa_2} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon-1} Y_t + \varepsilon \frac{MC_t(i)}{P_{H,t}} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon-1} Y_t \} \\ &\quad + \Theta \beta E_t \left[ \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} \left( \frac{P_{H,t+1}(i)}{P_{H,t}(i)} - 1 \right) \frac{P_{H,t+1}(i)}{P_{H,t}(i)^2} P_{H,t+1} \right], \\ \tau_{s,t} \omega_1 \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon(1-\omega_2)} Y_t^{1-\omega_2} &= \kappa_1 \kappa_2 U_t(i)^{\kappa_2-1} \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t. \end{aligned}$$

where where  $MC_t(i)$  is firm  $i$ 's marginal cost.

The symmetric equilibrium can be written as

$$\begin{aligned} \Theta \Pi_{H,t} (\Pi_{H,t} - 1) &= [(1 - \varepsilon) Y_t + \varepsilon \chi_2 \tau_{s,t} (1 - U_t) \omega_1 Y_t^{1-\omega_2} + \kappa_1 \varepsilon U_t^{\kappa_2} Y_t + \varepsilon MC_t Y_t] \\ &\quad + \beta \Theta E_t \left[ \frac{\Lambda_{t+1} \frac{(1-\theta+\theta \beta_{t+1}^{1-\varepsilon})^{-1}}{1-\varepsilon}}{\Lambda_t \frac{(1-\theta+\theta \beta_t^{1-\varepsilon})^{-1}}{1-\varepsilon}} (\Pi_{H,t+1} - 1) \Pi_{H,t+1} Y_{t+1} \right]. \end{aligned} \quad (28)$$

$$\tau_t Y_t^{-\omega_2} = \kappa_1 \kappa_2 U_t^{\kappa_2-1}. \quad (29)$$

where  $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$  is the domestic price index (DPI hereafter) inflation rate at time  $t$ .

Substituting  $\tau_{s,t} = \kappa_1 \kappa_2 U_t^{\kappa_2-1} Y_t^{\omega_2}$  into (28) yields the following New Keynesian Phillips curve (NKPC)

$$\begin{aligned} \pi_{H,t} (\pi_{H,t} - 1) &= \frac{\varepsilon}{\theta} \left[ \frac{(1-\varepsilon)}{\varepsilon} + \chi_2 \omega_1 \kappa_1 \kappa_2 U_t^{\kappa_2-1} (1 - U_t) + \kappa_1 U_t^{\kappa_2} + MC_t \right] \\ &\quad + \beta E_t \left[ \frac{\Lambda_{t+1} \frac{(1-\theta+\theta \beta_{t+1}^{1-\varepsilon})^{-1}}{1-\varepsilon}}{\Lambda_t \frac{(1-\theta+\theta \beta_t^{1-\varepsilon})^{-1}}{1-\varepsilon}} (\pi_{H,t+1} - 1) \pi_{H,t+1} \frac{Y_{t+1}}{Y_t} \right]. \end{aligned} \quad (30)$$

## Government

Suppose that the monetary authority sets its nominal interest rate  $R_t$  according to a generalized Taylor rule as in Clarida, Gali, and Gertler (1999, Clarida et al.):

$$R_t = R_{t-1}^{\rho_r} [Y_t^{b_y} \Pi_t^{b_\pi}]^{(1-\rho_r)} \exp(\varepsilon_{R,t}), \quad (31)$$

where  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the CPI inflation rate at time  $t$ , and  $\varepsilon_{R,t}$  is an i.i.d.  $N(0, \sigma_R^2)$ .

It is assumed that the fiscal authority levies lump taxes and emission by maintaining a balanced budget,  $P_t(T_t + \tau_t Y_t) = P_{H,t} G_t$ , where the (log) government spending  $G_t$  follows an AR (1) process:

$$\log G_t = \rho_G \log G_{t-1} + \varepsilon_{G,t}, \quad (32)$$

where  $\varepsilon_{G,t}$  is an i.i.d.  $N(0, \sigma_G^2)$ .

In this paper, we consider four environmental policy regimes. First one is a laissez-faire. Government does not implement any environmental policy, i.e.  $\tau_{e,t} = 0$ , implying  $X_t = 0$ . Second one is a cap regime wherein government imposes a fixed amount of emission  $e_t = \bar{e}$ . In this case  $\tau_{e,t}$  can be interpreted as the price of emission permits sold by the government, determined endogenously. Third one is targeting the amount of emission. Government imposes an emission target per unit of output  $e_t = \gamma Y_t$ , and it sells emission permits as in cap regime. Finally, we consider the taxation of government on emissions,  $\tau_{e,t}$ .

## Equilibrium

The equilibrium condition is given by

$$Y_t = (1 - \theta) S_t (1 - \theta + \theta S_t^{1-\psi})^{\frac{\psi}{1-\psi}} (C_t + I_t) + G_t + \theta S_t^\psi Y_t^*. \quad (33)$$

Because we focus on the symmetric equilibrium in which all agents in the same country make the same decisions, we need to define a symmetric equilibrium. The symmetric equilibrium conditions consist of (i) the efficiency conditions and the budget constraints of the domestic households and firms, (ii) the corresponding conditions of foreign countries, (iii) the risk-sharing condition, and (iv) market clearing conditions of each goods market, capital rental market, labor market, money, and bond market in each country, given the policies of monetary and fiscal authorities

## Quantitative Evaluation of the Model

### Parameter Values

All parameter values used in this paper are reported in Table 1. Most of them are taken from Annicchiarico and Di Dio (2015) and King and Watson (1996). The baseline model of this paper takes the intertemporal elasticity of consumption and the intratemporal elasticity of labor supply equal to 0.5 and 1. With this temporal utility function, the parameter values which will be used in the simulation can be determined. The serial correlation parameters for productivity, government spending, and markup shocks,  $\rho_A$ ,  $\rho_y^*$ ,  $\rho_g$  and  $\rho_\mu$  are assumed to be 0.9 as in Faia and Monacelli (2008), and Gal 13053'f and Monacelli (2005).

$$\begin{aligned} a_t &= 0.9a_{t-1} + \varepsilon_t^a, \quad \sigma_a = 0.01, \\ y_t^* &= 0.9y_{t-1}^* + \varepsilon_t^*, \quad \sigma_{y^*} = 0.01, \\ g_t &= 0.9g_{t-1} + \varepsilon_t^g, \quad \sigma_g = 0.01, \\ \mu_t &= 0.9\mu_{t-1} + \varepsilon_t^\mu, \quad \sigma_\mu = 0.01. \end{aligned}$$

As an interest rate smoothing rule, Clarida, Gali, and Gertler (1999)'s estimate for the Fed's monetary reaction function during Volcker-Greenspan era, as shown in the simulation below, is utilized.

$$r_t = 0.66r_{t-1} + 0.34(1.97E_t\pi_{t+1} + 0.07y_t) + \varepsilon_t^r, \quad (34)$$

where  $\varepsilon_t^r$  i.i.d. (0,0.0025). The parameter values concerning the investment adjustment cost are also taken from Smets and Wouters (2002). The investment adjustment cost function,  $\psi$ , is restricted to satisfy  $\psi(1) = \psi'(1) = 0$  and  $\psi''(1) > 0$  as in CEE (2001). The value of elasticity of  $i_t/i_{t-1}$  with respect to Tobin's  $q$ ,  $\eta_q$  which reflects the volatility of investment is set to 4 in the baseline model as in Smets and Wouters (2002). Finally, we set  $\epsilon$  to 6 so that the benchmark average sizes of price markup equal 1.2.

Turning to the calibration regarding emissions, we set the depreciation rate of emission  $\rho_x$  to 0.0021 as in Heutel (2012), implying that the emissions stock is depreciating at a rate of 0.8% per year. Foreign CO<sub>2</sub> emission  $e_t^*$  is assumed to be 1.33. Next, the coefficient  $\omega_1$ , measuring emissions per unit of output when there is no abatement effort ( $U_t = 0$ ) is set at 0.45 as in Annicchiarico and Di Dio (2015). The parameters in the damage function  $\Gamma(X_t)$ , i.e. ( $d_0$ ,  $d_1$ ,  $d_2$ ) are set as ( $1.3950 \times 10^{-3}$ ,  $-6.6722 \times 10^{-6}$ ,  $1.4647 \times 10^{-8}$ ). The initial stock of carbon dioxide in the atmosphere  $X$  is set at 800 which is consistent with the carbon mass of about 800 gigatons in 2005 as in Annicchiarico and Di Dio (2015).

In the environmental policy scenarios, we assume that government targets to reduce greenhouse gases by 20%. The parameters in the abatement cost function  $\kappa_1$  and  $\kappa_2$  are set to 0.1850 and 2.8 so as to have an abatement cost to output ratio equal to 0.15%.

**Table 1** The Calibrated Parameters

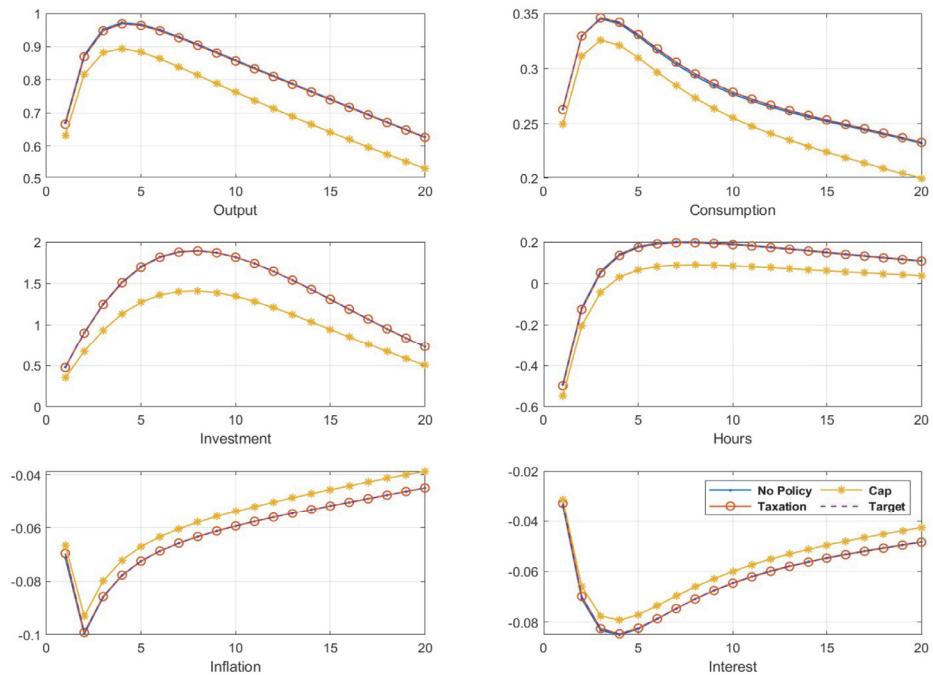
Parameter	Values	Description and definitions
$\epsilon_c(\sigma^{-1})$	0.5	Intertemporal elasticity of substitution
$s_H$	0.58	Steady state labor share
$\delta$	0.025	Rate of depreciation of capital stock
$r_h$	0.016	Steady state rate of return
$\theta$	0.4	Steady state share of imported consumption goods
$\alpha$	2/3	Fraction of firms that do not change their prices in a given period
$\epsilon$	6	Elasticity of demand for a good with respect to its own price
$\nu$	1	Inverse of elasticity of labor supply
$\eta_q$	1, 2, 4	Elasticity of $i/k$ to Tobin's $q$
$\rho_A, \rho_A^*$	0.9	First-order serial correlation of technology shock
$\rho_G$	0.9	First-order serial correlation of government shock
$\rho_\mu$	0.9	First-order serial correlation of markup shock
$\sigma_A, \sigma_{A^*}$	0.01	Standard Deviation of technology shock
$\sigma_\mu$	0.01	Standard Deviation of markup shock
$\sigma_r$	0.0025	Standard Deviation of monetary shock
$\sigma_G, \sigma_G^*$	0.01	Standard Deviation of government expenditure shock

Note: Country subscripts ( $h, f$ ) are suppressed. The same parameter values are used in the home country and the foreign country.

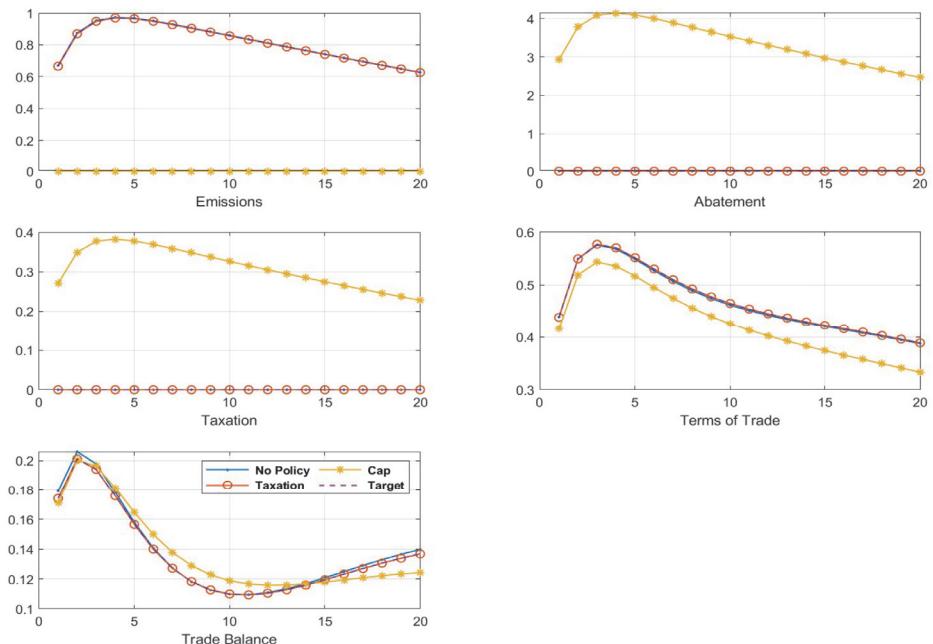
## Domestic productivity shock

In this subsection, we will discuss the dynamics of some selected variables to a favorable domestic productivity shock. To a favorable domestic productivity shock, households work less work, and firms increase their investments, yielding higher output and consumption and worsening trade balances. The real exchange rate depreciates to the domestic productivity shock as output expands to the shock. Prices fall to the domestic productivity shock, which calls for monetary policy authority to raise the interest rate to stabilize prices as in Figure 3A. In Figure, the solid lines ('-'), star lines ('-\*'), circled lines ('-o'), and long-dashed lines ('-') represent no environmental policy, cap, taxation, and intensity target regimes, respectively. There is little difference between no policy regime ( $\tau_{e,t} = 0$ ), target ( $e_t = \gamma Y_t$ ), and taxation ( $\tau_{e,t} = \tau_{ss} > 0$ ) regimes as in Figure 3B: Emission increases with output expansion, and constant abatement efforts.

Under the emissions cap wherein the taxation rate varies endogenously, there is no emission variation, but abatement effort increases with a higher taxation rate to control the emission. The response of relevant variables to the domestic productivity shock is more muted under an emission cap regime since higher costs occur to firms with convex abatement technology who have to keep the target emission levels fully. The more resources devoted to pollutant abatement, the more aggregate variables such as output, consumption, investment, and labor hours are muted.



**Fig. 3A** Impulse Response Function to a Domestic Productivity Shock



**Fig. 3B** Impulse Response Function to a Domestic Productivity Shock

## Government spending shock

The government spending expansion crowds out consumption and investment in the representative agent New Keynesian model. The rise in government spending induces households to work more because of the negative wealth effect. Since the aggregate demand contracts to the positive government spending shock, domestic price falls with a rise in government spending. The terms of trade appreciate a contraction of domestic consumption, which deteriorates trade balances as households divert their demand for goods toward foreign goods. Under this circumstance, the monetary authority lowers its policy rate to stabilize prices and boost the economy, as in Figure 4A.

Turning to the response of the environment-related variables displayed, domestic output increases with the rise in government spending, which translates into higher pollution emissions in all regimes except a cap regime. Figure 4B shows little difference between no policy regime, target, and taxation regimes. However, under the cap wherein the taxation rate varies endogenously, the emission is constant, and abatement varies with the taxation rate.

## Monetary policy shock

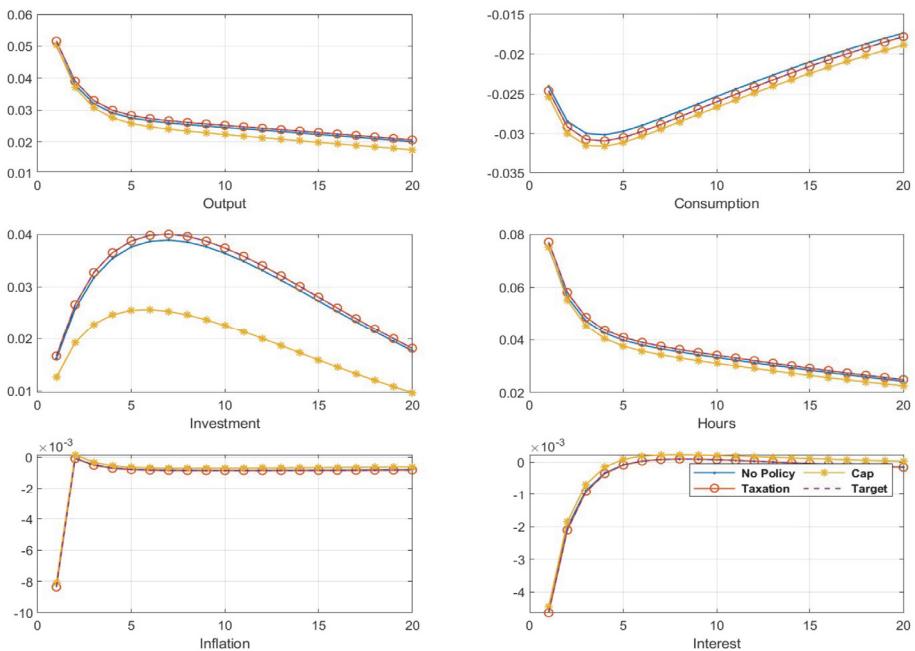
If the monetary authority lowers its policy rate, aggregate variables such as output, consumption, and investment expand, and prices rise. Under optimal risk-sharing, the increase in domestic consumption entails a real exchange rate depreciation, which improves trade balances in all regimes, as in Figure 5A. The expansion of aggregate output is translated into higher pollution emissions in all regimes except a cap regime.

Figure 5B shows little difference between no policy regime, target, and taxation regimes: Emission decreases with output contraction but constant abatement. Under the cap wherein the taxation rate varies endogenously, the emission is constant, and abatement varies with the taxation rate.

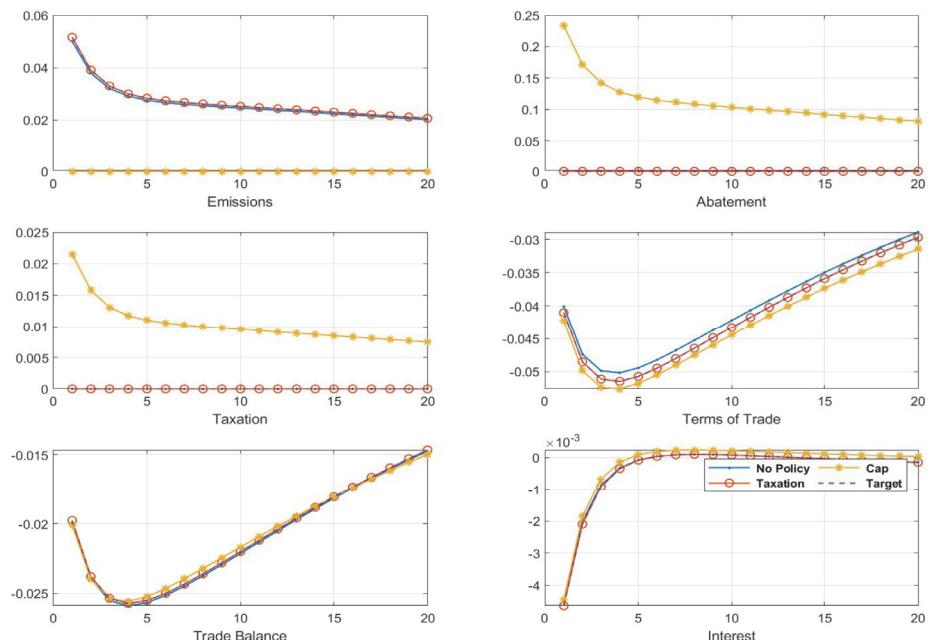
## Foreign productivity shock

The demand for domestic goods increases due to the favorable foreign productivity shock, which shifts the foreign aggregate demand. As a result, domestic production expands, which entails expanding domestic consumption and investment. Since foreign consumption rises more than domestic consumption due to the positive foreign productivity shock, the real exchange rate and the terms of trade appreciate, which diverts the demand for goods toward foreign goods, deteriorating trade balances as in Figure 6A.

The pollution emission dynamics resemble the aggregate output dynamics in all regimes except a cap regime. Figure 6B shows little difference between no policy regime, target, and taxation regimes: Emission increases with domestic output expansion but constant abatement. Under the cap wherein the taxation rate varies endogenously, the emission is constant, and abatement varies with the taxation rate.



**Fig. 4A** Impulse Response Function to a Domestic Government Spending Shock



**Fig. 4B** Impulse Response Function to a Domestic Government Spending Shock

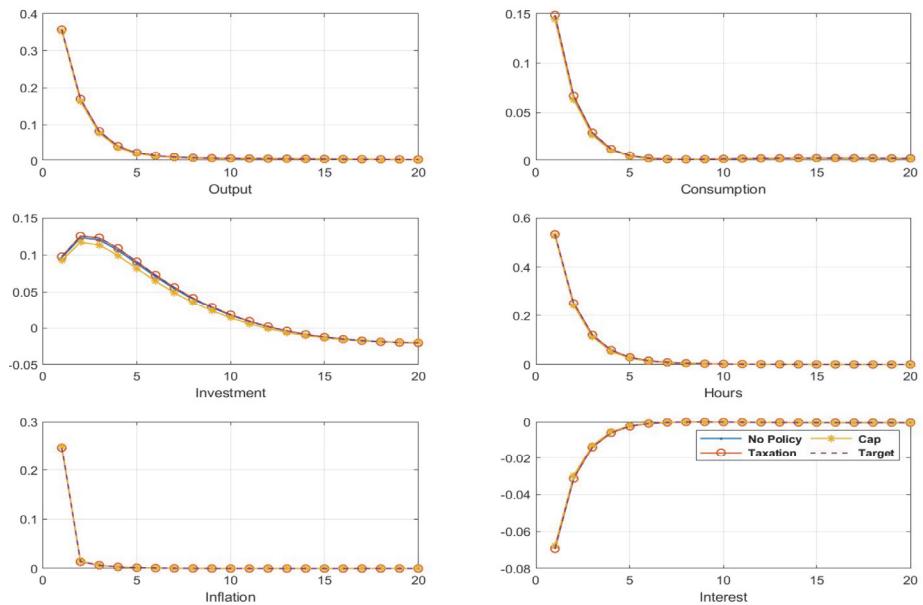


Fig. 5A Impulse Response Function to a Domestic Monetary Policy Shock

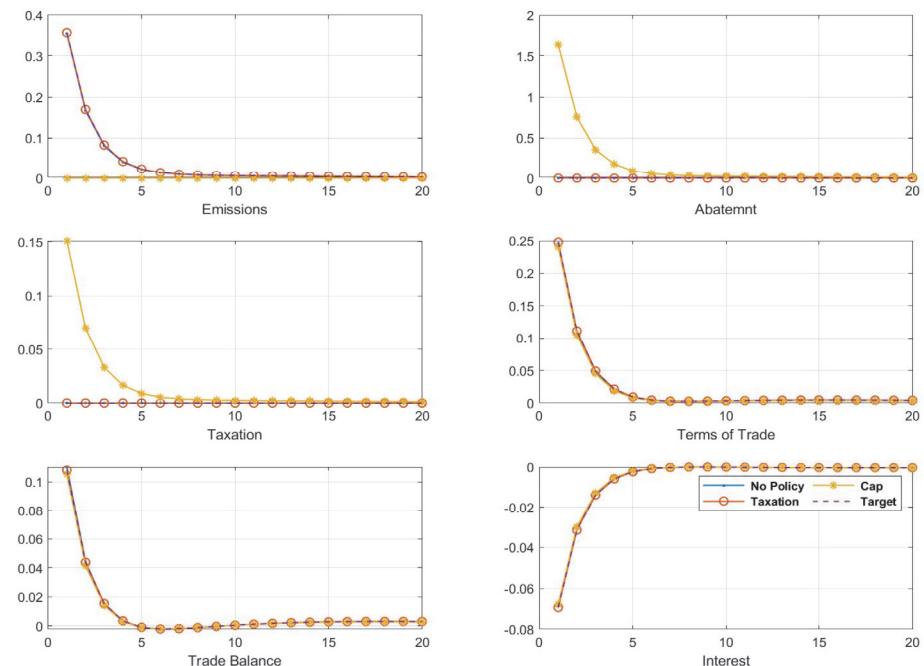
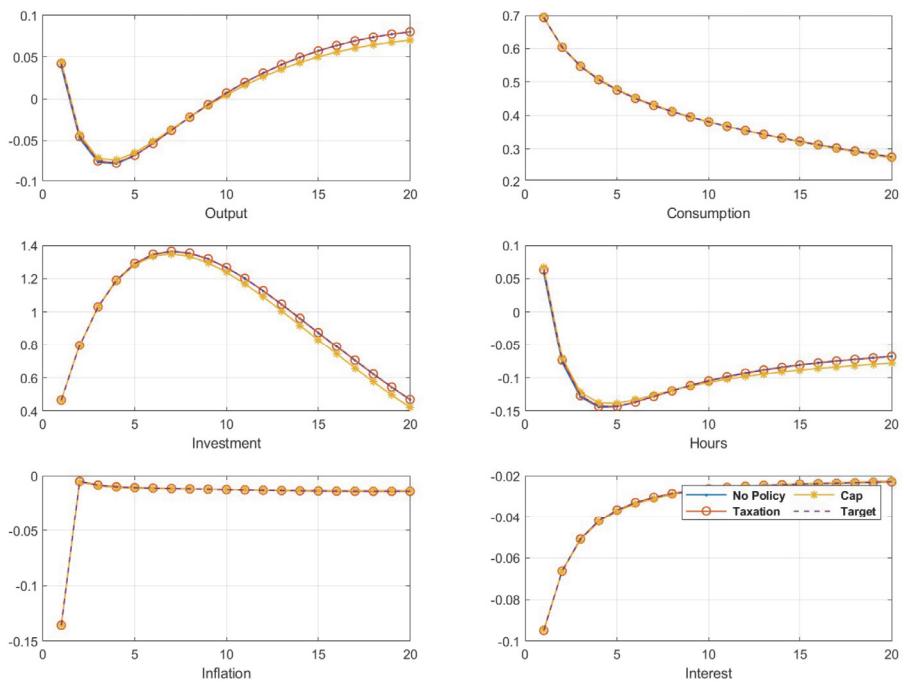
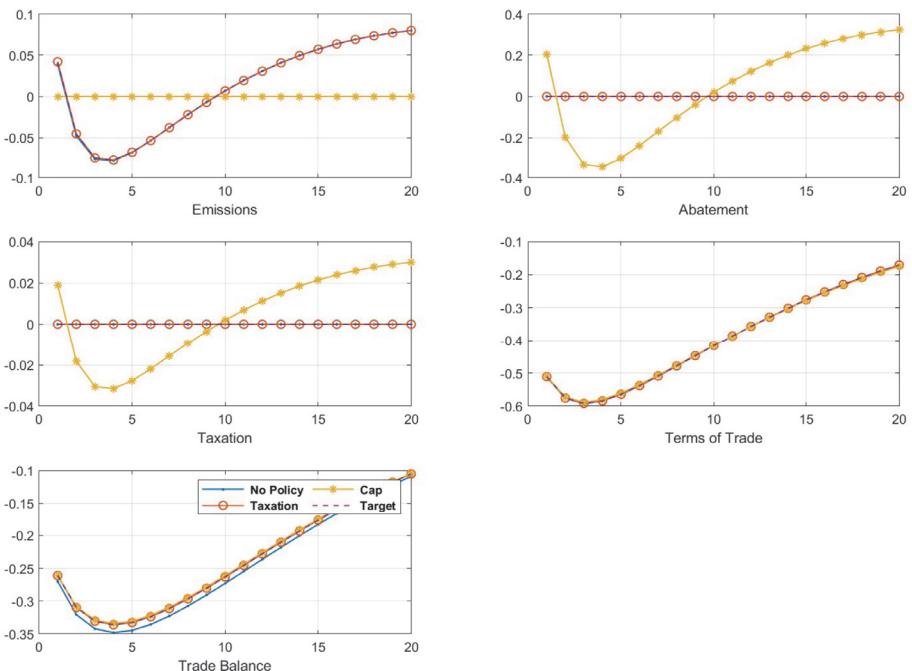


Fig. 5B Impulse Response Function to a Domestic Monetary Policy Shock



**Fig. 6A** Impulse Response Function to a Foreign Productivity Shock



**Fig. 6B** Impulse Response Function to a Foreign Productivity Shock

## Markup shock

Due to the unfavorable markup shock, the production cost of domestic firms increases. Hence, domestic output contracts, but prices increase, which entails a fall in domestic consumption, investment, and hours. The risk-sharing condition translates to an appreciation of the real exchange rate and the terms of trade. However, the domestic monetary authority increases its policy rate to stabilize prices, as in Figure 7A.

The contraction of aggregate output is translated into lower pollution emissions in all regimes except a cap regime. Figure 7B shows little difference between no policy regime, target, and taxation regimes: Emission decreases with output contraction but constant abatement. Under the cap wherein the taxation rate varies endogenously, emission is constant, but efforts of abatement decrease with a lower taxation rate.

## Optimal Progressive Taxations on Emission

In this subsection, we will consider a simple, implementable, optimal carbon tax rule as the Taylor interest rate rule. Suppose that taxations on emission (relative to the steady-state level)  $\frac{\tau_t}{\tau_{ss}}$  are proportional to emissions as follows:

$$\begin{aligned}\frac{\tau_{e,t}}{\tau_{ss}} &= \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[ \left(\frac{e_t}{e_{ss}}\right)^{\alpha_e} \left(\frac{Y_t}{Y_{ss}}\right)^{\alpha_y} \right]^{\rho_\tau}, \\ \frac{\tau_{x,t}}{\tau_{ss}} &= \left(\frac{\tau_{x,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[ \left(\frac{X_t}{X_{ss}}\right)^{\alpha_X} \left(\frac{Y_t}{Y_{ss}}\right)^{\alpha_y} \right]^{\rho_\tau}, \\ \frac{\tau_{y,t}}{\tau_{ss}} &= \left(\frac{\tau_{y,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[ \left(\frac{Y_t}{Y_{ss}}\right)^{\alpha_{y/X}} \right]^{\rho_\tau}, \\ \frac{\tau_{s,t}}{\tau_{ss}} &= \left(\frac{\tau_{s,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[ \left(\frac{X_t/X_{ss}}{Y_t/Y_{ss}}\right)^{\alpha_{X/Y}} \right]^{\rho_\tau},\end{aligned}$$

where  $\rho_\tau$  is tax smoothing parameter and  $\alpha_e$ ,  $\alpha_y$  are the elasticity of taxations to emissions and output and so on. Here  $X$  is the steady-state value of  $X_t$ . The parameters  $\alpha_e$ ,  $\alpha_{e/Y}$ ,  $\alpha_{X/Y}$  and  $\alpha_y$  are assumed to be positive since it is desirable to increase the tax rate on emissions to restore the target level of emission when the actual emission exceeds its target  $X(e)$  or the steady-state output. In an extreme situation,  $\alpha_e \rightarrow \infty$ , the macroeconomic consequences are expected to coincide with the capacity policy that is studied in the literature (Fischer and Springborn (2011)). In addition, if  $\alpha_e = 0$ , then the government simply implements a constant tax rate as in the previous subsection.

Table 2A - 2C presents welfare and policy coefficients associated with the optimized simple environmental taxation rules. It shows that the government should levy taxes on emissions, considering the effect of emission taxation on output. Since the emission stock in the economy is composed of controllable domestic pollution and uncontrollable foreign sources of emission,

the environmental taxation on domestic emission is far better than the environmental taxation based on the stock of total pollution. Furthermore, Table 2A - 2C displays that tax smoothing is not desirable to reduce emissions efficiently over time.

**Table 2A** Optimized Rule and Welfare

	$\tau_{e,t} = \tau_{ss}$	$\left(\frac{\tau_{e,t}}{\tau_{ss}}\right) = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left(\frac{e_t/e_{ss}}{X_t/X_{ss}}\right)^{\alpha_e(1-\rho_\tau)}$
Welfare	-103.9637	-103.9594
$\rho_\tau$	0.5	0
$\alpha_e$	2.4	3

**Table 2B** Optimized Rule and Welfare

	$\left(\frac{\tau_{e,t}}{\tau_{ss}}\right) = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left(\frac{e_t/e_{ss}}{Y_t/Y_{ss}}\right)^{\alpha_e(1-\rho_\tau)}$	$\left(\frac{\tau_{e,t}}{\tau_{ss}}\right) = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left(\frac{X_t/X_{ss}}{Y_t/Y_{ss}}\right)^{\alpha_e(1-\rho_\tau)}$
Welfare	-103.9637	-103.9637
$\rho_\tau$	0.5	0.4
$\alpha_e$	2.4	0

**Table 2C** Optimized Rule and Welfare

	$\left(\frac{\tau_{e,t}}{\tau_{ss}}\right) = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[\left(\frac{e_t}{e_{ss}}\right)^{\alpha_e} \left(\frac{Y_t}{Y_{ss}}\right)^{\alpha_y}\right]^{1-\rho_\tau}$	$\left(\frac{\tau_{e,t}}{\tau_{ss}}\right) = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[\left(\frac{X_t}{X_{ss}}\right)^{\alpha_y} \left(\frac{Y_t}{Y_{ss}}\right)^{\alpha_y}\right]^{1-\rho_\tau}$
Welfare	-103.9543	-103.9571
$\rho_\tau$	0	0
$\alpha_e$	3	3
$\alpha_y$	3	3

Figures 8A - 12B present the impulse response of some selected variables to each shock, when the government implements an optimized simple taxation rule for emissions  $\tau_{e,t} = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[\left(\frac{e_t/e_{ss}}{Y_t/Y_{ss}}\right)^{\alpha_e/Y}\right]^{\rho_\tau}$  and  $\tau_{e,t} = \left(\frac{\tau_{e,t-1}}{\tau_{ss}}\right)^{\rho_\tau} \left[\left(\frac{X_t/X_{ss}}{Y_t/Y_{ss}}\right)^{\alpha_X/Y}\right]^{\rho_\tau}$  for  $\rho_\tau = 0$  and  $\alpha_{e/Y} = \alpha_{X/Y} = 3$ . The dynamics of some selected variables to each specified shock under the optimized environmental taxation rule are similar to the ones under a taxation rule in previous section. Figures 8A - 12B show that the relevant variables show more muted impulse response to exogenous shocks under the optimized taxations on domestic emission flows than under the optimized taxations on total emission stock, which mirrors the welfare difference in Table 2A-2C.

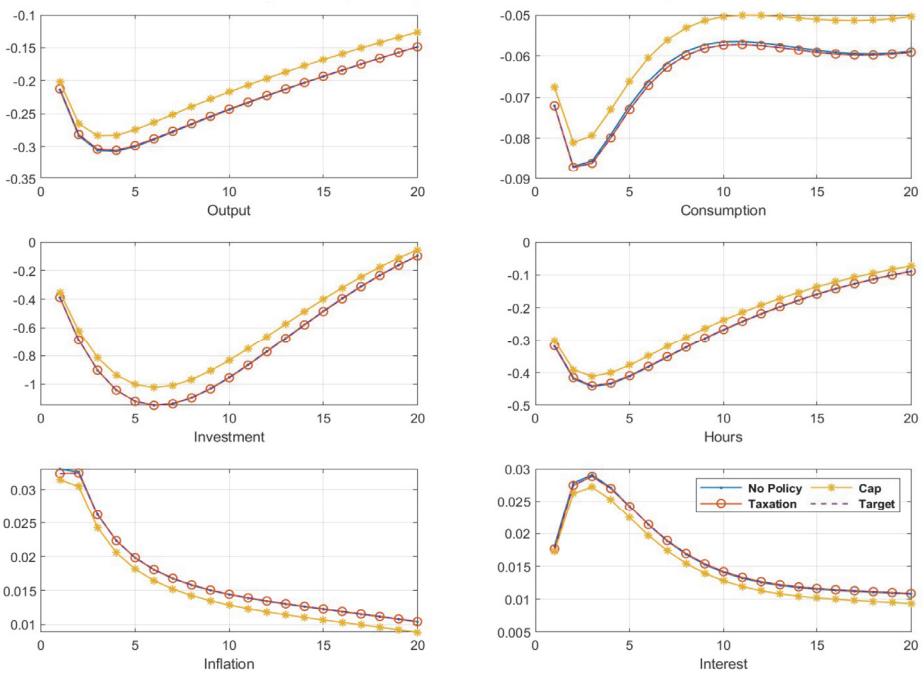


Fig. 7A Impulse Response Function to a Domestic Markup Shock

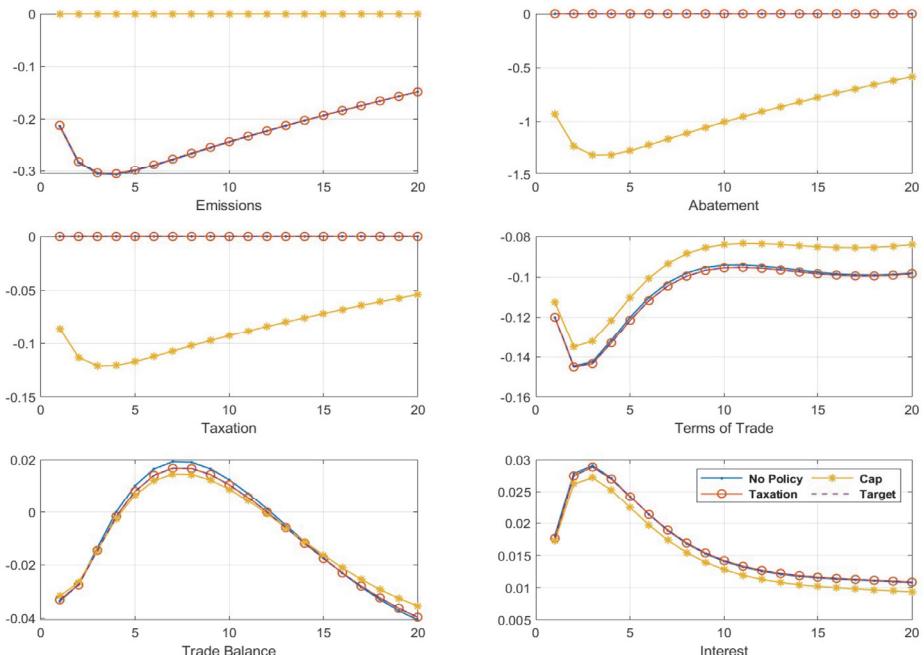
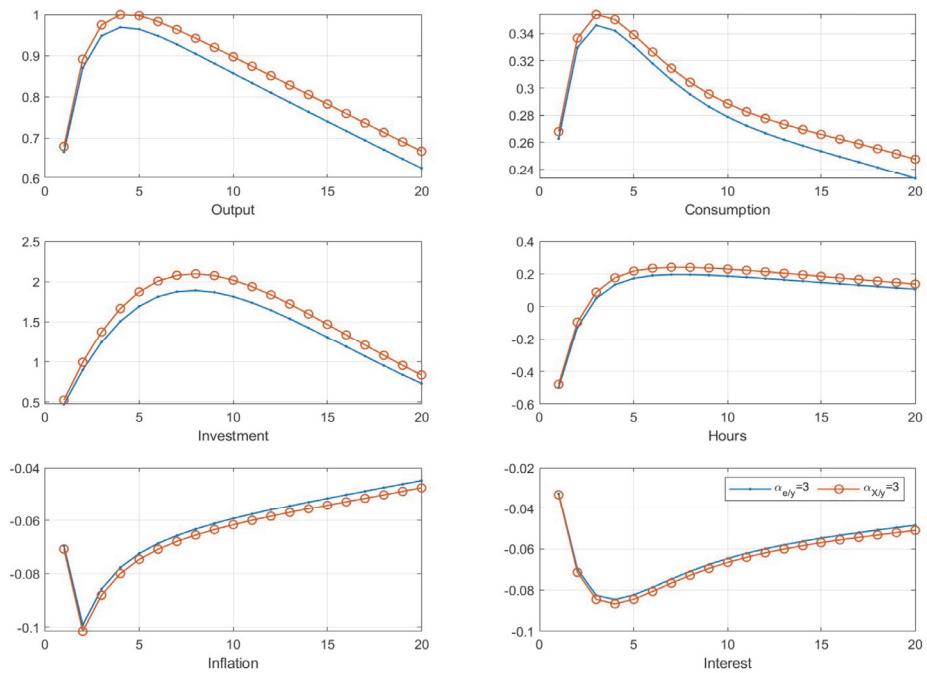
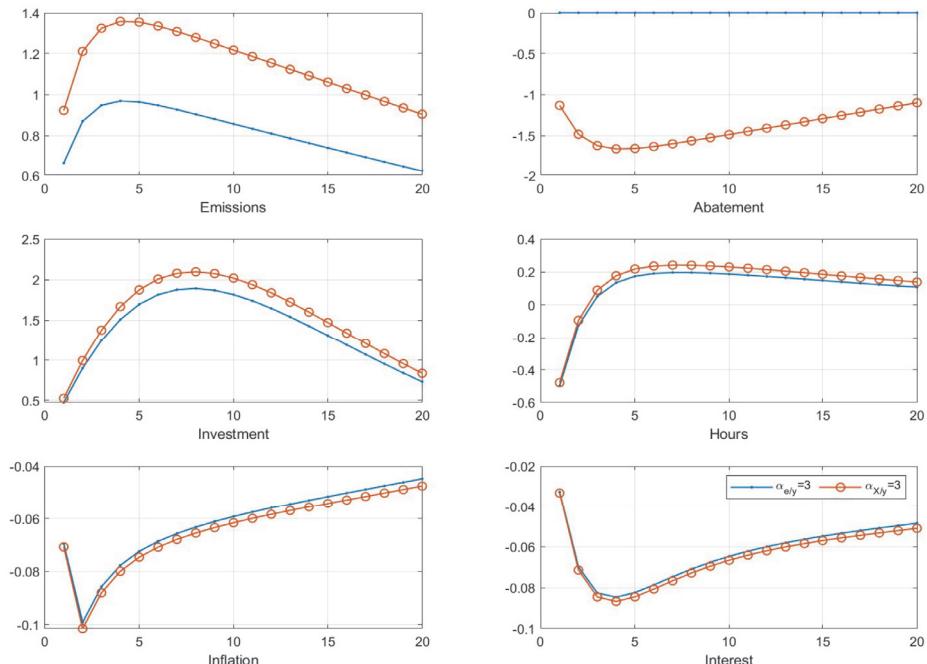


Fig. 7B Impulse Response Function to a Domestic Markup Shock



**Fig. 8A** Impulse Response Function to a Domestic Productivity Shock



**Fig. 8B** Impulse Response Function to a Domestic Productivity Shock

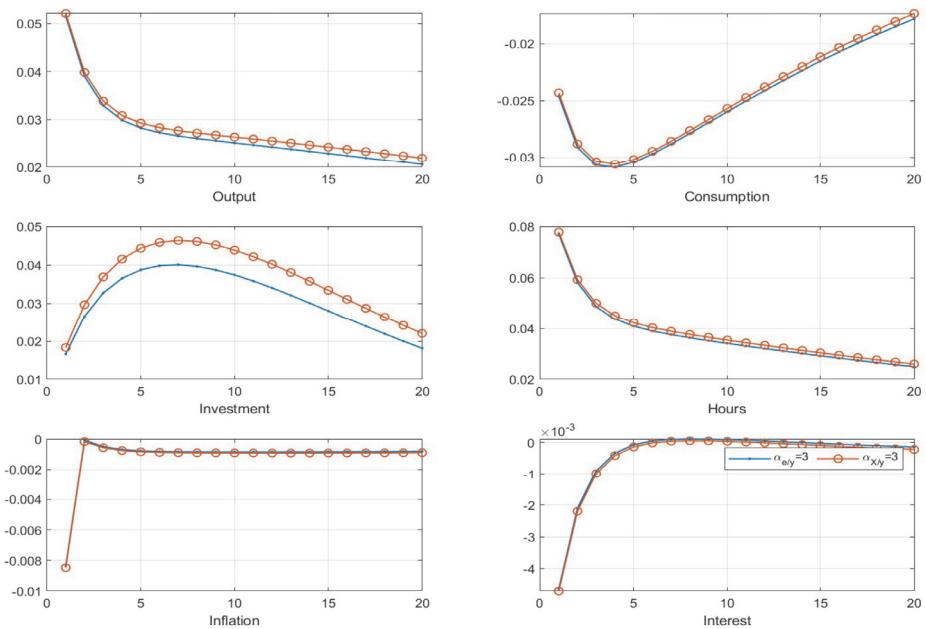


Fig. 9A Impulse Response Function to a Domestic Government Spending Shock

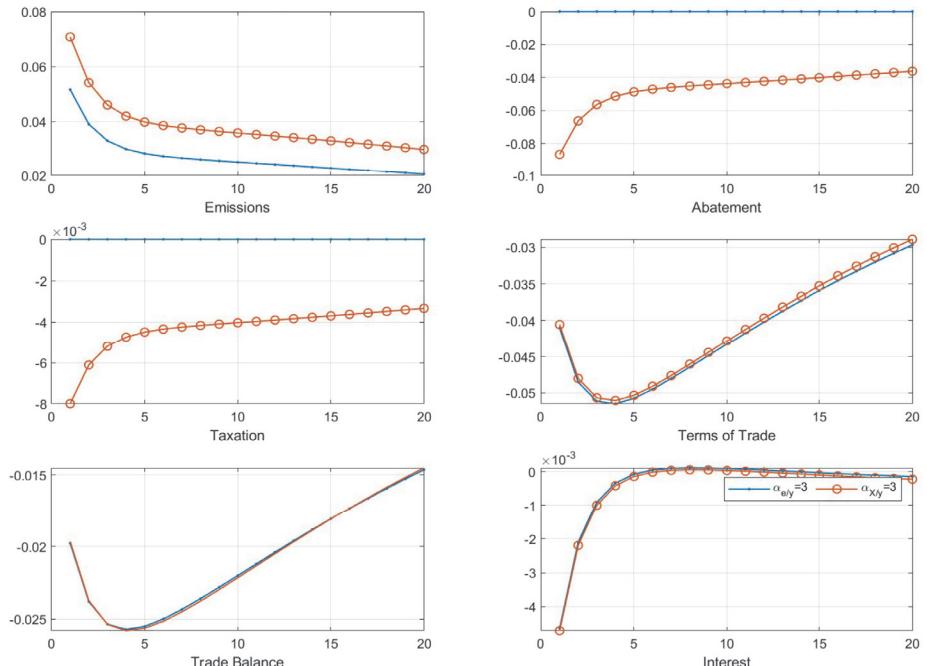
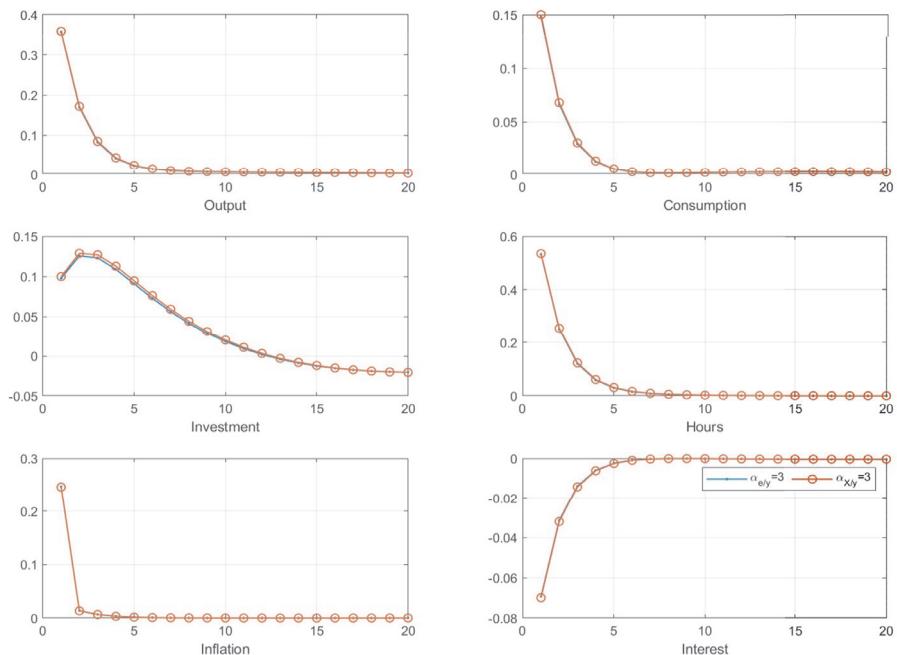
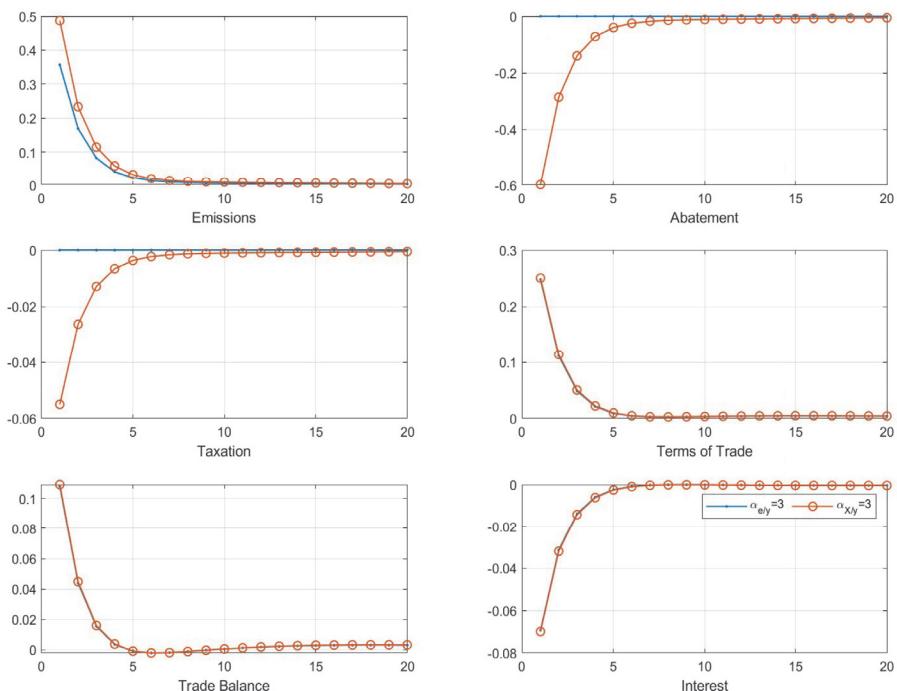


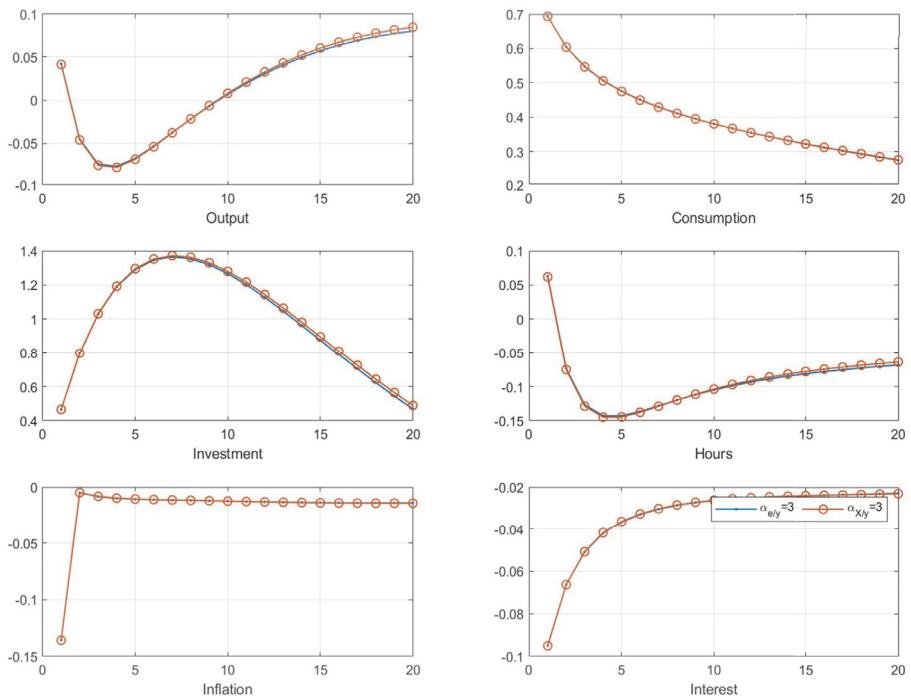
Fig. 9B Impulse Response Function to a Domestic Government Spending Shock



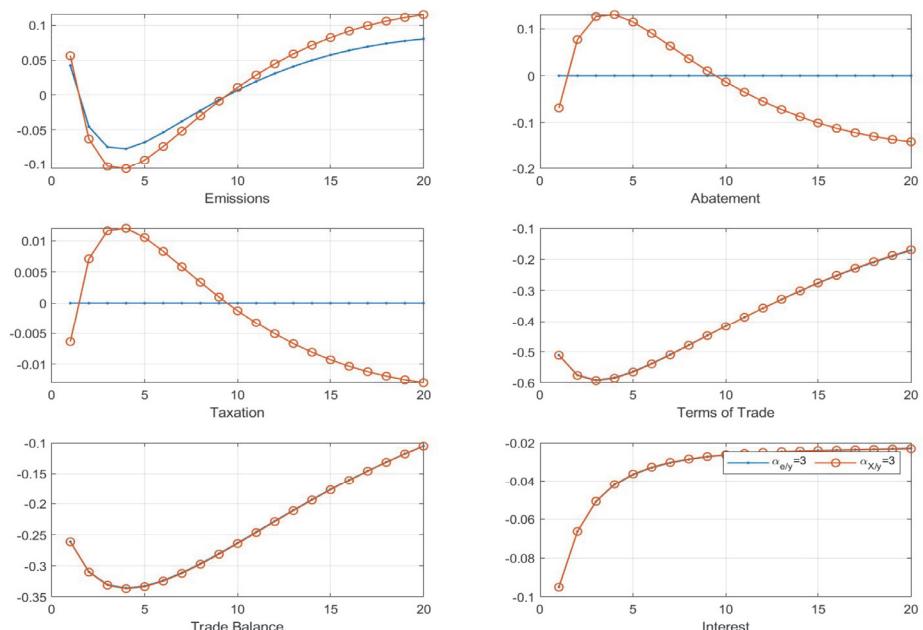
**Fig. 10A** Impulse Response Function to a Domestic Monetary Policy Shock



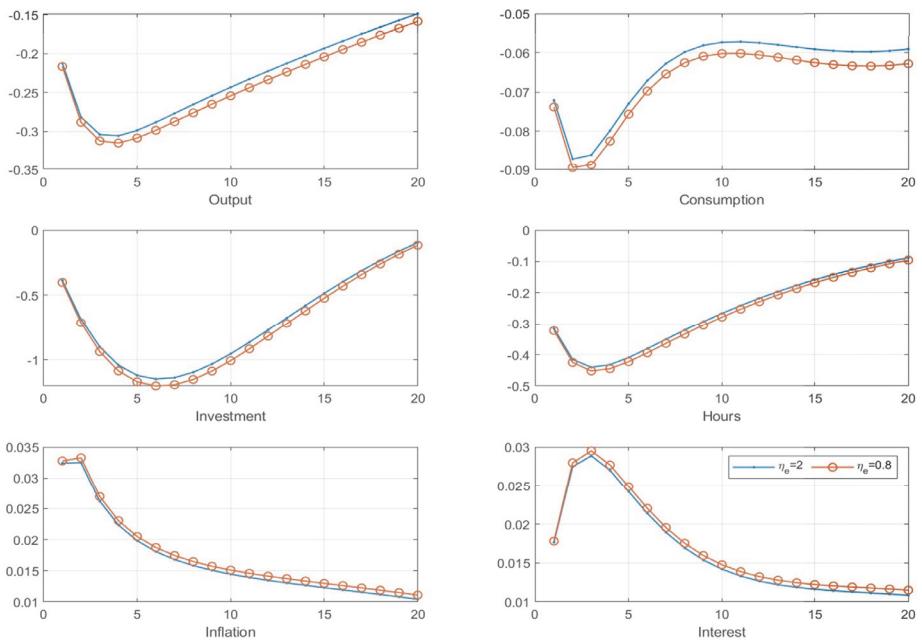
**Fig. 10B** Impulse Response Function to a Domestic Monetary Policy Shock



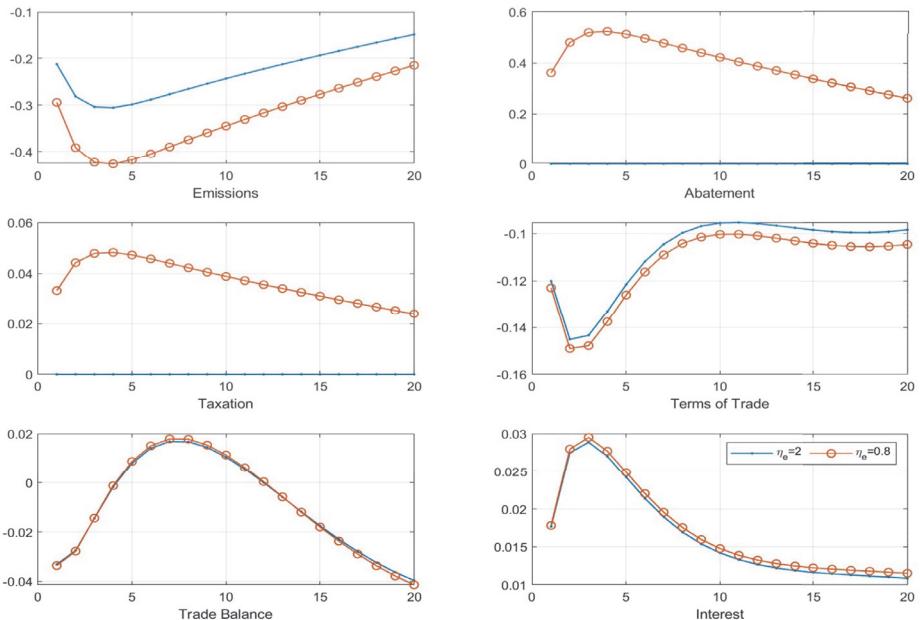
**Fig. 11A** Impulse Response Function to a Foreign Productivity Shock



**Fig. 11B** Impulse Response Function to a Foreign Productivity Shock



**Fig. 12A** Impulse Response Funciton to a Domestic Markup Shock



**Fig. 12B** Impulse Response Funciton to a Domestic Markup Shock

## Conclusion

This paper analyzed the impact of various environmental policies on the economy by including pollution emissions in the macroeconomic model. We have set up a canonical small open economy New Keynesian model embodying pollutant emissions and environmental policy. We have analyzed the performance of alternative environmental policy rules and found the optimal policy response to emissions. First, the simple optimized taxations should respond strongly to emissions and output since the emission is procyclical and firms' abatement efforts are procyclical, dampening business cycle fluctuations. Second, environmental taxation should be levied on domestic emission flows, not the overall stock of emissions, since it is more efficient and likely to generate less macroeconomic volatility by inducing firms to reduce emissions and optimally adjust their activities over time. Finally, the simple optimized environmental taxations show that the environmental taxation policy should be aggressive and not tax-smoothing type since the so-called tax-smoothing policy is welfare detrimental.

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